



Complex yield criteria for three-dimensional granular flows

Wolfgang Fellin (1) and Matthias Rauter (2,3)

(1) University of Innsbruck, Institute of Infrastructure, Department of Engineering Science, Innsbruck, Austria (wolfgang.fellin@uibk.ac.at), (2) Norwegian Geotechnical Institute, Department of Natural Hazards, Oslo, Norway (matthias.rauter@ngi.no), (3) University of Oslo, Department of Mathematics, Oslo, Norway

The incompressible non-Newtonian Navier-Stokes equations have become a popular method for the simulation of granular flows [1, 2, 3]. The success of the $\mu(I)$ -rheology [4] is certainly not insignificant for this. Within the framework of plasticity [5], this rheology can be described in terms of a yield surface and a flow rule. The yield surface is of Drucker-Prager type, parametrized with the inertial number I and the flow rule follows from incompressibility as von-Mises type.

However, the yield-strength of granular material depends on the loading type, e.g. plane strain shearing, triaxial extension or triaxial compression, which cannot be modelled with a Drucker-Prager yield criterion. Thus the Drucker-Prager yield criterion is no longer used in soil mechanics. The Mohr-Coulomb criterion or the Matsuoka-Nakai [6] criterion yield much more realistic results for the shearing failure of granular material. The difference to the Drucker-Prager criterion can be remarkable, especially if the loading type changes in time or space.

We demonstrate that the Drucker-Prager criterion within the $\mu(I)$ -rheology can be easily replaced with an arbitrary yield surface, using the usual assumption of alignment between stretching and deviatoric stress. We introduce the Matsuoka-Nakai yield criterion and thus obtain a rheology for granular flows that appropriately predicts different yield-strengths for different loading types. This rheology is, similar to the classic $\mu(I)$ -rheology, fully compatible with the incompressible Navier-Stokes Equations and can be implemented into CFD frameworks such as OpenFOAM [7].

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